Introduction to Bell's Theorem

Graduate Student Seminar

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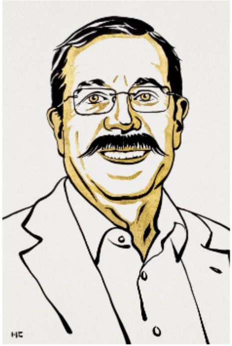
The curious trio

The Nobel Prize in Physics 2022



Ill. Niklas Elmehed © Nobel Prize Outreach

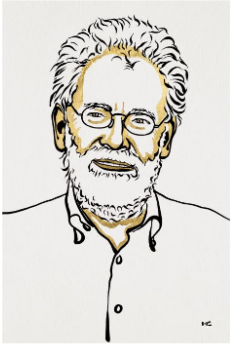
Alain Aspect

Prize share: 1/3

Ill. Niklas Elmehed © Nobel Prize Outreach

John F. Clauser

Prize share: 1/3



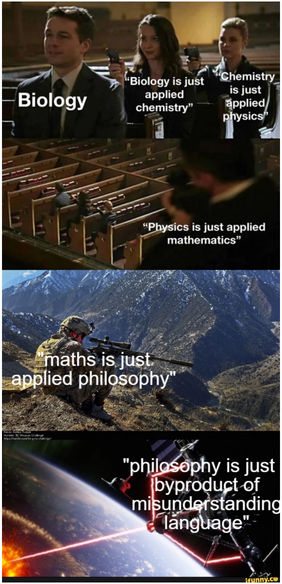
Ill. Niklas Elmehed © Nobel Prize Outreach

Anton Zeilinger

Prize share: 1/3

Picture: The Nobel Prize in Physics 2022. NobelPrize.org. Nobel Prize Outreach AB 2022. Thu. 3 Nov 2022.

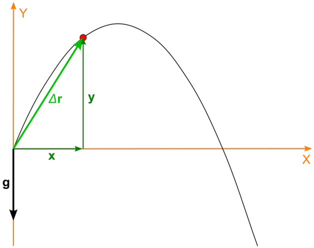
Outline


Biology
“Biology is just applied chemistry”
“Chemistry is just applied physics”
“Physics is just applied mathematics”
"maths is just applied philosophy"
"philosophy is just byproduct of misunderstanding language"
ifunny.co

Outline

- Mathematical Framework
- The EPR Paradox
- Bell's Inequality
- The experiments

States and Spaces



$$y(t) = v_0 \sin \alpha t - 0.5 g t^2$$
$$x(t) = v_0 \cos \alpha t$$
$$y = \tan \alpha x - \frac{1}{2 v_0^2 \cos^2 \alpha} x^2$$

Figure: By Zátonyi Sándor, (ifj.)
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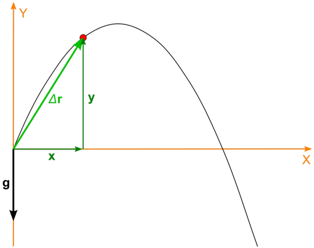
States and Spaces



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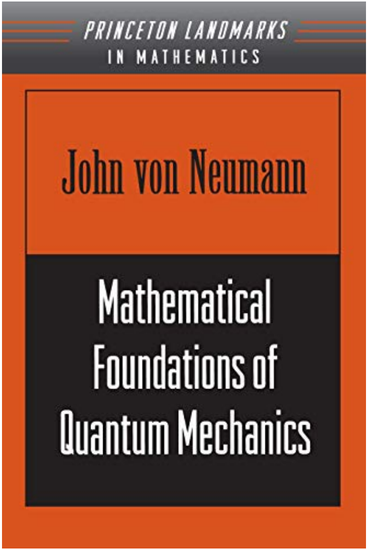
Each point on the curve is $(x, y) = x\hat{i} + y\hat{j} = x\,|i\rangle + y\,|j\rangle$

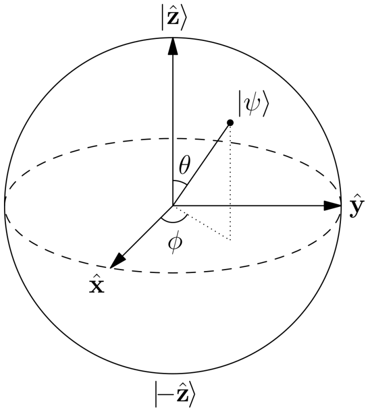
States $\iff$ points in a vector space

Hilbert Spaces



Picture: https://www.amazon.com/Mathematical-Foundations-Quantum-Mechanics-Neumann/dp/0691028931

Electron States

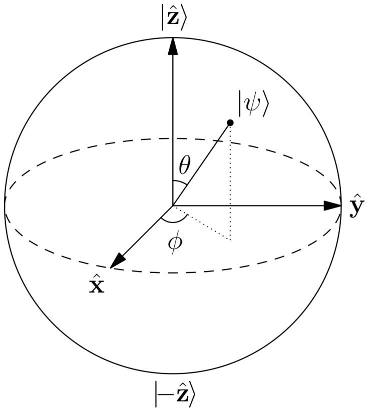


$$|\psi\rangle = \cos(\theta/2)\,|\hat{z}\rangle + e^{i\phi}\sin(\theta/2)\,|-\hat{z}\rangle$$
$$= \alpha\,|\hat{z}\rangle + \beta\,|-\hat{z}\rangle$$

where $\alpha^2 + \beta^2 = 1$

Figure:
https://en.wikipedia.org/wiki/File:Bloch_sphere.svg

Electron States



$$
\begin{aligned}
|\psi\rangle &= \cos(\theta/2)\,|\hat{z}\rangle + e^{i\phi}\sin(\theta/2)\,|-\hat{z}\rangle \\
&= \alpha\,|\hat{z}\rangle + \beta\,|-\hat{z}\rangle
\end{aligned}
$$

where $\alpha^2 + \beta^2 = 1$

Figure:
https://en.wikipedia.org/wiki/File:Bloch_sphere.svg

Each point on the sphere is $(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\phi)$

Multiple Electrons

Building States

$|\psi_1\rangle \otimes |\psi_2\rangle \otimes ... \otimes |\psi_n\rangle$

Multiple Electrons

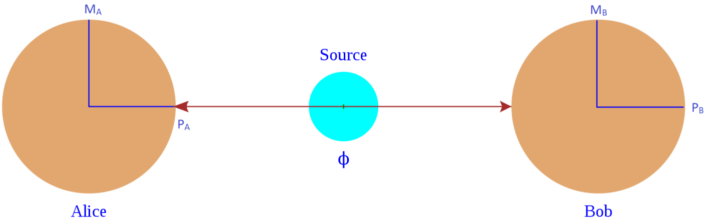
Building States

$|\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_n\rangle$

Entangled Particles

$|\psi_1\rangle \otimes |\psi_2\rangle \neq \alpha\, |\widehat{\mathbf{z}}\rangle \otimes |-\widehat{\mathbf{z}}\rangle + \beta\, |-\widehat{\mathbf{z}}\rangle \otimes |\widehat{\mathbf{z}}\rangle$
$|\psi_1\rangle \otimes |\psi_2\rangle \neq \alpha\, |\widehat{\mathbf{x}}\rangle \otimes |-\widehat{\mathbf{x}}\rangle + \beta\, |-\widehat{\mathbf{x}}\rangle \otimes |\widehat{\mathbf{x}}\rangle$
where $\widehat{\mathbf{z}}$ represents $\uparrow$, $-\widehat{\mathbf{z}}$ represents $\downarrow$, $\widehat{\mathbf{x}}$ represents $\longrightarrow$ and $-\widehat{\mathbf{x}}$ represents $\longleftarrow$

EPR Paradox



Adopted from `https://en.wikipedia.org/wiki/EPR_paradox#/media/File:EPR_illustration.svg`

EPR Paradox Bohm

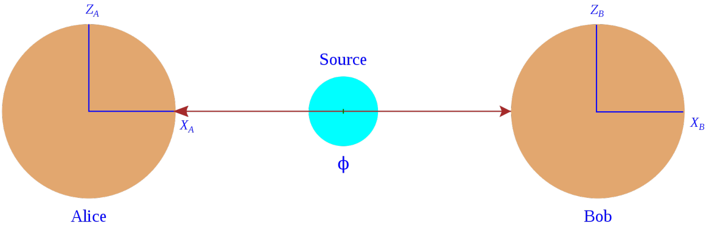
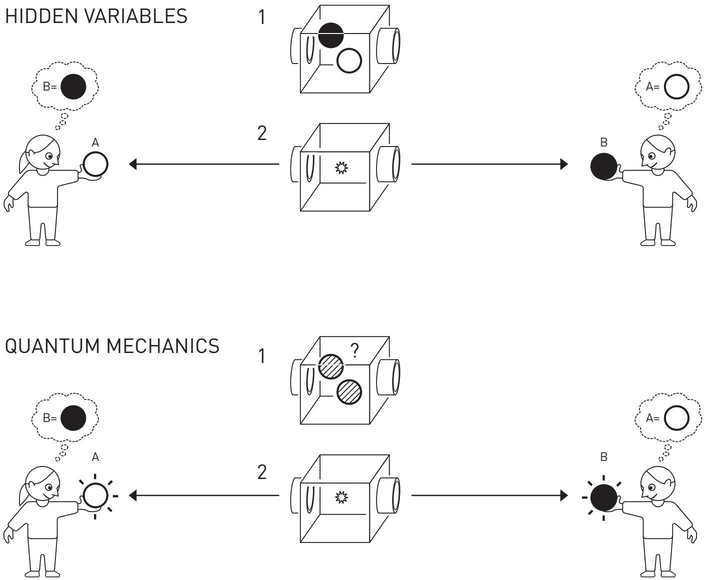


Image from https://en.wikipedia.org/wiki/EPR_paradox#/media/File:EPR_illustration.svg

Note: $\widehat{\mathbf{z}} = \uparrow$, $-\widehat{\mathbf{z}} = \downarrow$, $\widehat{\mathbf{x}} = \longrightarrow$ and $-\widehat{\mathbf{x}} = \longleftarrow$
and $|\widehat{\mathbf{z}}\rangle = \frac{|\widehat{\mathbf{x}}\rangle + |-\widehat{\mathbf{x}}\rangle}{\sqrt{2}}$ and $|-\widehat{\mathbf{z}}\rangle = \frac{|\widehat{\mathbf{x}}\rangle - |-\widehat{\mathbf{x}}\rangle}{\sqrt{2}}$

Is the moon not there when no one looks at it?



Bell's Inequality

$$\mu(A \cap B^c) \leq \mu(A \cap H^c) + \mu(H \cap B^c)$$

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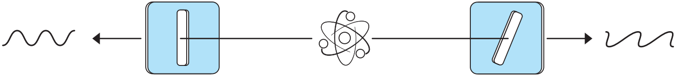
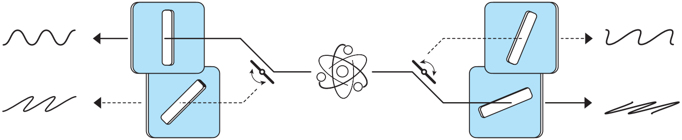
Proof Sketch

$$\begin{aligned} A \cap B^c &= A \cap \mathcal{U} \cap B^c \\ &= A \cap (H \cup H^c) \cap B^c \\ &= (A \cap H) \cup (A \cap H^c) \cap B^c \\ &= (A \cap H \cap B^c) \cup (A \cap H^c) \\ &\subset (A \cap H^c) \cup (H \cap B^c) \end{aligned}$$

The actual experiments

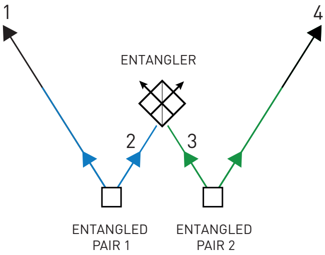


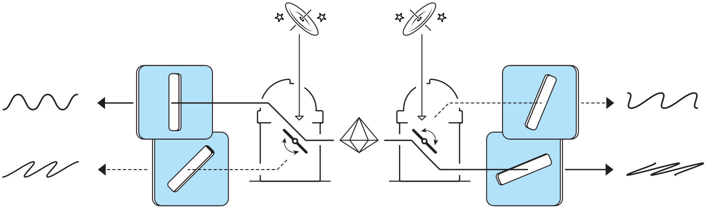
John Clauser used calcium atoms that could emit entangled photons after he had illuminated them with a special light. He set up a filter on either side to measure the photons' polarisation. After a series of measurements, he was able to show they violated a Bell inequality.



Alain Aspect developed this experiment, using a new way of exciting the atoms so they emitted entangled photons at a higher rate. He could also switch between different settings, so the system would not contain any advance information that could affect the results.

The actual experiments





Anton Zeilinger later conducted more tests of Bell inequalities. He created entangled pairs of photons by shining a laser on a special crystal, and used random numbers to shift between measurement settings. One experiment used signals from distant galaxies to control the filters and ensure the signals could not affect each other.